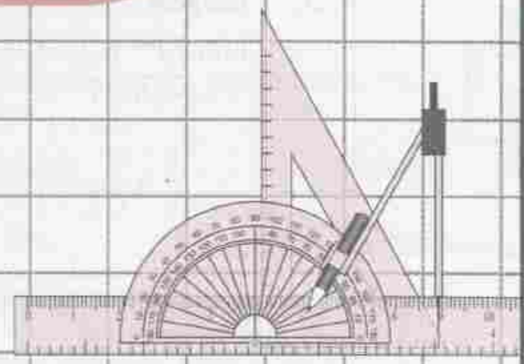
CHAPTER 1

# Number Systems



## In This Chapter

- Number Systems
- Four Fundamental Operations
- Rational Numbers
- Properties of Fundamental Operations

## Starter

Match the given set of numbers with the number systems to which they belong.

| Numbers | Number System |
|---|---|
| … –3, –2, –1, 0, 1, 2, 3…. | Whole numbers |
| 1, 2, 3, 4…. | Integers |
| 0, 1, 2, 3, 4…. | Natural numbers |

Let us recall the three basic number systems with the help of the table given below.

| S. No. | Number System | Description |
|---|---|---|
| 1. | Natural (or Counting) numbers<br>N = {1, 2, 3, ...} | • All numbers 1, 2, 3, 4, ... are called **natural numbers**.<br>• The number 1 is the smallest natural number and there is no greatest natural number. |
| 2. | Whole numbers<br>W = {0, 1, 2, 3, ...} | • The set of natural numbers along with 0 forms a new set of numbers called **whole numbers**.<br>• The set of whole numbers contain all natural numbers.<br>• The number 0 is the smallest whole number and there is no greatest whole number.<br>• If we multiply a whole number by 0, the product is always 0.<br>• If we divide 0 by a whole number other than 0, the quotient is always 0. However, if we divide a whole number other than 0 by 0, the quotient cannot be determined.<br>• When any whole number (other than 0) is divided by itself, the quotient is always 1. |

| | | |
|---|---|---|
| 3. | Integers<br>I (or Z) = {..., –2, –1, 0, 1, 2, 3, ...} | • The set of whole numbers and the negative values of all natural numbers form a new set of numbers called **integers**.<br>• The set of integers contain the set of whole numbers and additive inverse of all natural numbers.<br>• The set of integers contain all whole numbers. |

*Try These*

1. Compare the following numbers by putting < or > sign in the boxes.
   (i) –3 ☐ 0 (ii) –24 ☐ –30
   (iii) –39 ☐ 26 (iv) 29 ☐ –13
2. Arrange the following integers in ascending order.
   (i) –12, 0, 15, 21, –26, –5
   (ii) –17, 14, –12, 10, –6, 0
3. Add the following integers on a number line.
   (i) (–2) and 2 (ii) 5 and (–4)
   (iii) (–3) and (–6)

## Rational Numbers (Q)

We know that a temperature of $5\frac{1}{2}$ degrees above 0°C is shown as $+5\frac{1}{2}$. But what about a temperature which is $5\frac{1}{2}$ degrees below zero? Can you write it as $-5\frac{1}{2}$ or $\frac{-11}{2}$? For this, we need to study about a new group of numbers called rational numbers.

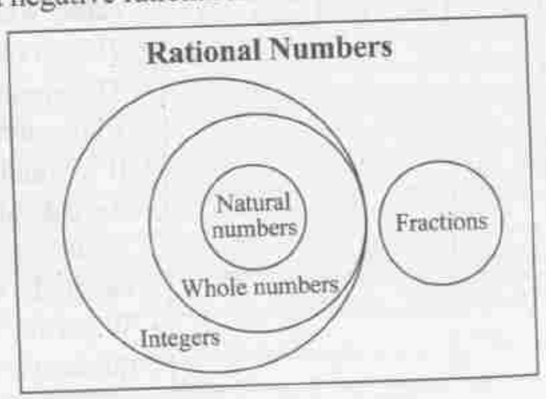
**Rational numbers** are numbers which can be expressed in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$. $p$ is called the **numerator** of the rational number and $q$ is called its **denominator**. The word rational comes from the word ratio which is basically a comparison by division. We know that ratios can be expressed as fractions, for example, 3 : 4 is the same as $\frac{3}{4}$.

It is interesting to note that the set of all natural numbers, whole numbers and integers are included in the set of rational numbers.

We can write the numbers 3, 0 and –7 as $\frac{3}{1}$, $\frac{0}{1}$ and $\frac{-7}{1}$ without changing their values. As they are in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$, they are rational numbers. Rational numbers also include fractions and decimals. We already know that decimals can also be written in the form of fractions, for example, $0.25 = \frac{25}{100}$.

### Positive and negative rational numbers

Like integers, rational numbers can also be positive or negative. Let us look at some rational numbers $\frac{2}{3}, \frac{-4}{7}, \frac{-15}{-21}$ and $\frac{16}{-41}$. Here, in the given rational numbers $\frac{2}{3}$ and $\frac{-15}{-21}$, both the numerator and the denominator are either positive or negative. So, $\frac{2}{3}$ and $\frac{-15}{-21}$ are positive rational numbers. But $\frac{-4}{7}$ and $\frac{16}{-41}$ have a negative sign either in the numerator or in the denominator. Hence, they are negative rational numbers. Zero is neither a positive nor a negative rational number.






**Quick Tip**

If both numerator and denominator are negative, it is a positive rational number.

### Standard form of rational numbers

In the standard form of a rational number,

1. the rational number is expressed in its lowest terms.
2. the denominator of the rational number is expressed as a positive integer.

For example,

(i) Let us write $\frac{-16}{-18}$ in the standard form.

Here, the HCF of 16 and 18 is 2.

So, $\frac{-16}{-18} = \frac{(-16) \div 2}{(-18) \div 2} = \frac{-8}{-9}$

But in the standard form the denominator has to be positive.

Thus, $\frac{-8}{-9} = \frac{(-8) \times (-1)}{(-9) \times (-1)} = \frac{8}{9}$

(ii) Let us write $\frac{14}{-21}$ in the standard form.

Here, the HCF of 14 and 21 is 7.

So, $\frac{14}{-21} = \frac{14 \div 7}{(-21) \div 7} = \frac{2}{-3}$

But the denominator has to be positive.

Thus, $\frac{2}{-3} = \frac{2 \times (-1)}{(-3) \times (-1)} = \frac{-2}{3}$

## Solved Examples

***Example 1:*** Write the following as rational numbers.

(i) –108 (ii) 2.76

***Solution:***

(i) $\frac{-108}{1}$ (ii) $\frac{2.76}{1} = \frac{276}{100}$

***Example 2:*** Identify the positive and negative rational numbers from the following rational numbers.

(i) $\frac{-7}{-11}$ (ii) $\frac{2}{-13}$ (iii) $\frac{-2}{15}$ (iv) $\frac{5}{21}$

***Solution:***

(i) Positive (ii) Negative
(iii) Negative (iv) Positive

***Example 3:*** Write the following rational numbers in standard form.

(i) $\frac{50}{-100}$ (ii) $\frac{-17}{119}$ (iii) $\frac{-24}{-576}$ (iv) $\frac{13}{169}$

***Solution:***

(i) $\frac{50}{-100} = \frac{-50}{100} = \frac{-5}{10} = \frac{-1}{2}$

(ii) $\frac{-17}{119} = \frac{-1}{7}$

(iii) $\frac{-24}{-576} = \frac{24}{576} = \frac{1}{24}$

(iv) $\frac{13}{169} = \frac{1}{13}$

## Exercise 1.1

1. Write the following as rational numbers.
   (i) 18 (ii) –17 (iii) –438
   (iv) 0.05 (v) 2.38 (vi) 3.8
2. Identify the positive rational numbers from the following rational numbers.
   (i) $\frac{7}{-8}$ (ii) $\frac{9}{-11}$ (iii) $\frac{-3}{-7}$
   (iv) $\frac{12}{-19}$ (v) $\frac{3}{5}$ (vi) $\frac{-5}{13}$
3. Write any ten positive rational numbers.
4. Write any ten negative rational numbers.

5. Identify the negative rational numbers from the following rational numbers.

(i) $\frac{7}{-11}$ (ii) $\frac{-13}{-16}$ (iii) $\frac{-100}{5}$

(iv) –12 (v) $\frac{17}{-6}$ (vi) $\frac{-2}{-7}$

6. Write the following rational numbers in standard form.

(i) $\frac{-16}{-20}$ (ii) $\frac{12}{-36}$ (iii) $\frac{100}{-200}$

(iv) $\frac{-7}{63}$ (v) $\frac{11}{-121}$ (vi) $\frac{-23}{-138}$

## Four Fundamental Operations

We know that addition, subtraction, multiplication and division are called the four fundamental operations.

In previous classes, we have learnt to perform these operations on the set of natural and whole numbers. Let us now learn how to perform these operations on the set of integers and rational numbers.

### Integers

Let us now learn the rules for addition, subtraction, multiplication and division of integers.

**Addition of integers**

Let us learn the rules for addition of integers.

1. If the integers have the same sign, we add their absolute value and assign the same sign to the answer.

   For example,

   (i) (–5) + (–2) = –7

   (ii) (–7) + (–6) = –7 – 6 = –13

2. If the integers have the opposite signs, that is, one is positive and the other is negative, we find the difference and assign the sign of the greater number.

   For example,

   (i) –5 + 2 = –3 (ii) 8 + (–3) = 5

### Solved Examples

*Example 4:* Add the following integers.

(i) 9 and –5 (ii) –17 and 10

(iii) –4 and –21 (iv) –37 and –49

*Solution:*

(i) 9 + (–5) = 4 (ii) –17 + 10 = –7

(iii) –4 + (–21) = –25 (iv) –37 + (–49) = –86

*Example 5:* Evaluate the following.

(i) 4 + (–19) (ii) –16 + 14

(iii) –23 + (–27) (iv) –114 + (–26)

*Solution:*

(i) 4 + (–19) = –15 (ii) (–16) + 14 = –2

(iii) (–23) + (–27) = –50 (iv) (–114) + (–26) = –140

**Additive inverse of integers**

Two integers whose sum is zero are known as **additive inverse** of each other. They are also called the negative of each other. Additive inverse of an integer is obtained by changing the sign of the integer. For example, the additive inverse of +2 is –2 and the additive inverse of –8 is +8.

**Subtraction of integers**

Let us now learn the rules for subtraction of integers.

1. Find the additive inverse (or opposite) of the integer to be subtracted.
2. Add the additive inverse found in Step 1 to the other integer.

   For example, subtract 4 from –6.

   That is, find the value of –6 – 4.

   Additive inverse of 4 is –4.

   Add –4 to –6.

   –6 + (–4) = –10

   Therefore, –6 – 4 = –10.



### Solved Examples

*Example 6:* Subtract the following integers.

(i) 49 from –5 (ii) –27 from 11

(iii) –44 from –21 (iv) –35 from –49

*Solution:*

(i) (–5) – 49 = (–5) + (–49) = –54

(ii) 11 – (–27) = 11 + 27 = 38

(iii) (–21) – (–44) = –21 + 44 = 23

(iv) (–49) – (–35) = (–49) + 35 = –14

*Example 7:* Evaluate the following.

(i) (–4) – (–19) (ii) (–61) – 14

(iii) (–26) + (–37) – 14 (iv) (–120) + (–62) – (–16)

*Solution:*

(i) (–4) – (–19) = (–4) + 19 = 15

(ii) (–61) – 14 = (–61) + (–14) = –75

(iii) (–26) + (–37) – 14 = (–63) + (–14) = –77

(iv) (–120) + (–62) – (–16) = (–182) + 16 = –166

*Example 8:* What should be added to (–12) to get 3?

*Solution:* Let us take a simple example. What should be added to 2 to get 5? We know that the answer is 3. How did we get it? By subtracting 2 from 5.

Similarly, in our question, we need to subtract (–12) from 3.

So,

3 – (–12) = 3 + 12 = 15

**Check:** If we add 15 to (–12), we get 3.

Thus, 15 is added to (–12) to get 3.

**Multiplication of integers**

Now let us learn the rules for multiplication of integers.

1. To multiply two integers having same sign, we multiply their absolute values and assign positive sign to the product.

   For example,

   (i) 3 × 8 = 24 (ii) –5 × –6 = 30

2. To multiply two integers having opposite signs, we multiply their absolute values and assign negative sign to the product.

   For example,

   (i) –8 × 4 = –32 (ii) 12 × –3 = –36

### Solved Examples

*Example 9:* Find the product of the following integers.

(i) 2 and (–4) (ii) (–7) and 11

(iii) (–4) and (–16) (iv) (–3) and (–9)

*Solution:*

(i) 2 × (–4) = –8 (ii) (–7) × 11 = –77

(iii) (–4) × (–16) = 64 (iv) (–3) × (–9) = 27

*Example 10:* Evaluate the following.

(i) (–5) × (–19)

(ii) (–7) × 21

(iii) (–12) × {(–9) – (–1)}

(iv) {(–6) + (–7)} × {(–14) – 4}

*Solution:*

(i) (–5) × (–19) = 95

(ii) (–7) × 21 = –147

(iii) (–12) × {(–9) – (–1)} = (–12) × (–8) = 96

(iv) {(–6) + (–7)} × {(–14) – 4} = (–13) × (–18) = 234

## Division of integers

There are some rules to divide two integers which are as follows:

1. To divide two integers having same sign, we divide their absolute values and assign positive sign to the quotient.

   For example,

   (i) $45 \div 9 = 5$

   (ii) $-63 \div (-7) = 9$

2. To divide two integers having opposite signs, we divide their absolute values and assign negative sign to the quotient.

   For example,

   (i) $-84 \div 12 = -7$

   (ii) $96 \div (-4) = -24$

### *Quick Tip*

Whenever there are more than one operations, we use the BODMAS rule where B stands for Bracket, O for Of, D for Division, M for Multiplication, A for Addition and S for Subtraction, carried out in that order.

### *Did You Know*

The rules for multiplication and division of integers were put forward by the Indian Mathematician Brahmagupta in his work 'Brahmasputhasiddhanta' about 1400 years ago. He had claimed that positive times positive and negative times negative are positive. He also said that a negative divided by a negative is positive and a positive divided by a negative is negative.

## Solved Examples

***Example 11:*** Divide the following integers.

(i) 63 by (–9) (ii) (–26) by 13

(iii) (–490) by (–10) (iv) (–303) by (–3)

***Solution:***

(i) $63 \div (-9) = -7$

(ii) $(-26) \div 13 = -2$

(iii) $(-490) \div (-10) = 49$

(iv) $(-303) \div (-3) = 101$

***Example 12:*** Evaluate the following.

(i) $\{(-18) \div 3\} - 42$

(ii) $\{(-48) \div (-16)\} - \{42 \div (-21)\}$

(iii) $\{(-21) - 6\} \div \{(-49) + 40\}$

(iv) $\{56 + (-42)\} \div \{(-14) + 7\}$

***Solution:***

(i) $\{(-18) \div 3\} - 42 = -6 - 42 = -48$

(ii) $\{(-48) \div (-16)\} - \{42 \div (-21)\} = 3 - (-2) = 5$

(iii) $\{(-21) - 6\} \div \{(-49) + 40\} = (-27) \div (-9) = 3$

(iv) $\{56 + (-42)\} \div \{(-14) + 7\} = 14 \div (-7) = -2$

## Exercise 1.2

1. Add the following integers.

   (i) $-6 + (-2)$ (ii) $7 + (-11)$

   (iii) $-3 + 8$ (iv) $18 + (-16)$

   (v) $3 + 5 + (-4)$ (vi) $-2 + (-9)$

2. Subtract the following integers.

   (i) –3 from 7 (ii) 4 from (–3)

   (iii) 2 from (–11) (iv) 11 from (–16)

   (v) –9 from (–1) (vi) –4 from (–3)

3. Find the value of the following.

   (i) $14 + (-3) + 11$

   (ii) $-9 - 7 + (-6)$

   (iii) $15 - (-6) + (-7)$

   (iv) $4 + (-11) + (-6) - (-2)$

   (v) $-3 - (-7) + (-5)$

   (vi) $6 + 8 + (-12) - (-4)$



4. Put >, < or = sign in the boxes.

   (i) $-9 + (-7) + 1$ ☐ $(-8) + 15$

   (ii) $14 - 3 - 10$ ☐ $(-7) - 6$

   (iii) $(-3) + 18$ ☐ $7 + 8 + 0$

   (iv) $5 - 9 + (-3)$ ☐ $6 - 11 - 12$

   (v) $8 - 11$ ☐ $(-7) - (-4)$

5. Evaluate the following.

   (i) $(-3) \times (-2) \times 11$ (ii) $7 \times 5 \times (-3)$

   (iii) $(-2) \times (-4) \times (-7)$ (iv) $(-5) \times 4 \times (-2)$

   (v) $-3 \times 11 \times 6$ (vi) $8 \times (-3) \times 2$

6. Evaluate the following.

   (i) $\{(-7) \times (-6)\} \div \{2 \times (-7)\}$

   (ii) $\{(-12) \times 4\} \div \{(-3) \times (-8)\}$

   (iii) $\{(-5) \times (-3) \times 2\} \div \{(-5) \times (-1)\}$

   (iv) $\{(-8) - 100 + 12\} \div \{(-4) \times 3\}$

   (v) $\{110 \div 11\} \div \{(-2) \times (-5)\}$

   (vi) $\{49 + 63 - 14\} \div \{(-1) \times 7\}$

7. Add the sum of (–57) and (–13) to the sum of (–110) and 46.

8. Subtract the sum of (–23) and 54 from the sum of (–11), (–13) and (–21).

9. The product of three integers is (–110). If two of the integers are 11 and (–5), find the third integer.

10. Divide (–1000) by (–50) and to the quotient add the product of (–13) and (–7).

## Rational numbers

We have learnt in the previous classes how to add and subtract fractions. We have also learnt how to add and subtract integers. The rules for addition and subtraction of rational numbers are a combination of the rules for fractions and the rules for integers.

### Addition of rational numbers with same denominator

1. Write the given rational numbers in their standard form.
2. Add the numerators and retain the same denominator.
3. Write the answer in the standard form.

For example,

(i) $\frac{3}{11} + \frac{4}{11} = \frac{(3+4)}{11} = \frac{7}{11}$

(ii) $\frac{2}{5} + \frac{(-1)}{5} = \frac{2+(-1)}{5} = \frac{2-1}{5} = \frac{1}{5}$

### Addition of rational numbers with different denominators

1. Write the given rational numbers in their standard form.
2. Find the lowest common multiple (LCM) of the denominators of the two given rational numbers.
3. Now, express the given rational numbers such that LCM becomes their same denominator.
4. Add the numerators and retain the same denominator.
5. Write the answer in the standard form.

For example,

(i) Add $\frac{1}{6}$ and $\frac{-1}{3}$.

LCM of 6 and 3 is 6.

To express the given rational number with denominator 6, multiply both numerator and denominator by 2.

$\frac{-1}{3} = \frac{-1 \times 2}{3 \times 2} = \frac{-2}{6}$

Therefore, $\frac{1}{6} + \frac{-1}{3} = \frac{1}{6} + \frac{-2}{6} = \frac{1+(-2)}{6} = \frac{-1}{6}$

(ii) Add $\frac{-1}{5}$ and $\frac{2}{9}$.

Here, the denominators of the given rational numbers are 5 and 9 respectively.

LCM of 5 and 9 as 45.

Let us express the denominators of the given rational numbers as 45.

$\frac{-1}{5} = \frac{-1 \times 9}{5 \times 9} = \frac{-9}{45}$

$\frac{2}{9} = \frac{2 \times 5}{9 \times 5} = \frac{10}{45}$

Therefore, $\frac{-1}{5} + \frac{2}{9} = \frac{-9}{45} + \frac{10}{45} = \frac{-9+10}{45} = \frac{1}{45}$

### Additive inverse of a rational number

Just as in the case of integers, additive inverse of a rational number is the rational number of the same magnitude as the given rational number but with the opposite sign.

For example, the additive inverse of $\frac{-6}{11}$ is $\frac{6}{11}$ and of $\frac{3}{5}$ is $\frac{-3}{5}$.

Let us see what happens when we add two rational numbers which are additive inverse of each other.

(i) $\frac{6}{11}+\frac{-6}{11}=\frac{6+(-6)}{11}=\frac{0}{11}=0$

(ii) $\frac{-3}{5}+\frac{3}{5}=\frac{-3+3}{5}=\frac{0}{5}=0$

In both the cases, we get 0 as the answer.

Like integers, when we add additive inverse of rational numbers, we get 0 as answer.

## SOLVED EXAMPLES

***Example 13:*** Add the following.

(i) $\frac{7}{8}+\frac{(-3)}{8}$ (ii) $\frac{-4}{11}+\frac{-6}{-11}$

***Solution:***

(i) Here, we see that the rational numbers are in the standard form and the denominators are the same. So, we can add the numerators.

Thus, $\frac{7}{8}+\frac{(-3)}{8}=\frac{7+(-3)}{8}=\frac{4}{8}=\frac{1}{2}$

(ii) Writing $\frac{-6}{-11}$ in the standard form as $\frac{6}{11}$.

Thus, $\frac{-4}{11}+\frac{-6}{-11}=\frac{-4}{11}+\frac{6}{11}=\frac{-4+6}{11}=\frac{2}{11}$

***Example 14:*** Add the following.

(i) $\frac{-7}{12}+\frac{2}{3}$ (ii) $\frac{-1}{5}+\frac{3}{-4}$

***Solution:***

(i) Here, since the denominators are not the same, we need to convert both the rational numbers into like rational numbers.

LCM of the denominators, that is, 12 and 3 is 12.

So, $\frac{-7}{12}$ remains as it is.

$\frac{2}{3}=\frac{2\times4}{3\times4}=\frac{8}{12}$

Thus, $\frac{-7}{12}+\frac{2}{3}=\frac{-7}{12}+\frac{8}{12}=\frac{-7+8}{12}=\frac{1}{12}$

(ii) Here, also we have to convert the rational numbers into like rational numbers.

$\frac{-1}{5}+\frac{3}{-4}=\frac{-1}{5}+\frac{-3}{4}$

LCM of the denominators, that is, 5 and 4 is 20.

So, $\frac{-1}{5}=\frac{-1\times4}{5\times4}=\frac{-4}{20}$

and $\frac{-3}{4}=\frac{-3\times5}{4\times5}=\frac{-15}{20}$

Thus, $\frac{-1}{5}+\frac{-3}{4}=\frac{-4}{20}+\frac{-15}{20}$

$=\frac{(-4)+(-15)}{20}=\frac{-19}{20}$

### Subtraction of rational numbers

Now let us learn the rules for subtraction of rational numbers.

1. Write the rational numbers in their standard form.
2. Add the additive inverse of the rational number to be subtracted to the other rational number.
3. Follow the rules of addition of rational numbers while adding the additive inverse.

For example,

(i) $\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}=\frac{3}{7}$

(ii) $\frac{7}{9}-\frac{(-2)}{9}=\frac{7+2}{9}=\frac{9}{9}=1$

(iii) $\frac{(-2)}{5}-\frac{1}{10}=\frac{-4-1}{10}=\frac{-5}{10}=\frac{-1}{2}$



## SOLVED EXAMPLES

***Example 15:*** Subtract the following.

(i) $\frac{4}{9}$ from $\frac{-3}{9}$ (ii) $\frac{1}{-11}$ from $\frac{-6}{11}$

***Solution:***

(i) $\frac{-3}{9}-\frac{4}{9}$

The additive inverse of $\frac{4}{9}$ is $\frac{-4}{9}$.

Thus, we can write $\frac{-3}{9}-\frac{4}{9}=\frac{-3}{9}+\frac{-4}{9}$

$=\frac{(-3)+(-4)}{9}=\frac{-7}{9}$

(ii) $\frac{-6}{11}-\frac{1}{-11}=\frac{-6}{11}-\frac{-1}{11}$

The additive inverse of $\frac{-1}{11}$ is $\frac{1}{11}$.

Thus, $\frac{-6}{11}-\frac{-1}{11}=\frac{-6}{11}+\frac{1}{11}=\frac{-6+1}{11}=\frac{-5}{11}$

***Example 16:*** Find the value of the following.

(i) $\frac{6}{4}+\frac{3}{-5}-\frac{8}{10}$ (ii) $\frac{1}{-6}+\frac{1}{3}-\frac{3}{12}$

***Solution:***

(i) LCM of 4, 5 and 10 is 20.

So, $\frac{6}{4}+\frac{3}{-5}-\frac{8}{10}=\frac{6}{4}+\frac{-3}{5}+\frac{-8}{10}$

$=\frac{30}{20}+\frac{-12}{20}+\frac{-16}{20}$

$=\frac{30+(-12)+(-16)}{20}$

$=\frac{30-28}{20}=\frac{2}{20}=\frac{1}{10}$

(ii) LCM of 6, 3 and 12 is 12.

So, $\frac{1}{-6}+\frac{1}{3}-\frac{3}{12}=\frac{-1}{6}+\frac{1}{3}+\frac{-3}{12}$

$=\frac{-2}{12}+\frac{4}{12}+\frac{-3}{12}=\frac{(-2)+4+(-3)}{12}$

$=\frac{(-5)+4}{12}=\frac{-1}{12}$

***Example 17:*** The sum of two rational numbers is $\frac{-7}{12}$. If one of the numbers is $\frac{-2}{3}$, find the other.

***Solution:*** The other rational number will be $\frac{-7}{12}-\frac{-2}{3}$.

Now, $\frac{-7}{12}-\frac{-2}{3}=\frac{-7}{12}+\frac{2}{3}=\frac{-7}{12}+\frac{8}{12}=\frac{-7+8}{12}=\frac{1}{12}$

(LCM of 12 and 3 is 12.)

Let us check.

$\frac{-2}{3}+\frac{1}{12}=\frac{-8}{12}+\frac{1}{12}=\frac{-8+1}{12}=\frac{-7}{12}$

Thus, the other rational number is $\frac{1}{12}$.

***Example 18:*** What should be added to $\frac{6}{-11}$ so that the sum is $\frac{2}{3}$?

***Solution:*** The number to be added to $\frac{6}{-11}$

$=\frac{2}{3}-\frac{6}{-11}=\frac{2}{3}-\frac{-6}{11}=\frac{2}{3}+\frac{6}{11}$

$=\frac{22}{33}+\frac{18}{33}=\frac{22+18}{33}=\frac{40}{33}$

(LCM of 3 and 11 is 33.)

**Check:**

$\frac{6}{-11}+\frac{40}{33}=\frac{-6}{11}+\frac{40}{33}=\frac{-18}{33}+\frac{40}{33}$

$=\frac{-18+40}{33}=\frac{22}{33}=\frac{2}{3}$

Thus, the number to be added to $\frac{6}{-11}$ is $\frac{40}{33}$.

***Example 19:*** The sum of three rational numbers is $\frac{-6}{5}$. If two of the numbers are $\frac{3}{10}$ and $\frac{-9}{15}$, find the third number.

***Solution:*** The third number will be

$$\frac{-6}{5}-\left(\frac{3}{10}+\frac{-9}{15}\right)=\frac{-6}{5}-\left(\frac{9}{30}+\frac{-18}{30}\right)$$

(LCM of 10 and 15 is 30.)

$$=\frac{-6}{5}-\left(\frac{9+(-18)}{30}\right)=\frac{-6}{5}-\left(\frac{-9}{30}\right)$$

$$=\frac{-6}{5}+\frac{9}{30}=\frac{-36}{30}+\frac{9}{30}$$

(LCM of 5 and 30 is 30.)

$$=\frac{-36+9}{30}=\frac{-27}{30}=\frac{-9}{10}$$

Let us check.

$$\frac{3}{10}+\frac{-9}{15}+\frac{-9}{10}=\frac{9}{30}+\frac{-18}{30}+\frac{-27}{30}$$

$$=\frac{9+(-18)+(-27)}{30}$$

$$=\frac{9-45}{30}=\frac{-36}{30}=\frac{-6}{5}$$

Thus, the third rational number is $\frac{-9}{10}$.

## Multiplication of rational numbers

We know that the rules for addition and subtraction of rational numbers were a combination of the rules of addition and subtraction of fractions and integers. Similarly, the rules for multiplication of rational numbers are also a combination of the rules of multiplication of fractions and integers. Let us learn about these rules.

1. Write the rational numbers in their standard form.
2. Multiply the numerators by using the rules for multiplication of integers.
3. Multiply the denominators by using the rules for multiplication of integers.
4. Product will be $\frac{\text{Answer of step 2}}{\text{Answer of step 3}}$.
5. Write the product in the standard form.

For example,

(i) $\frac{-2}{5}\times\frac{6}{-11}=\frac{-2}{5}\times\frac{-6}{11}=\frac{2\times6}{5\times11}=\frac{12}{55}$

(ii) $\frac{3}{8}\times\frac{(-12)}{15}=\frac{3\times(-12)}{8\times15}=\frac{-36}{120}=\frac{-3}{10}$

## Solved Examples

***Example 20:*** Find the product of the following.

(i) $\frac{-7}{14}\times\frac{4}{-8}$ (ii) $\frac{16}{-20}\times\frac{3}{-9}$

***Solution:***

(i) $\frac{-7}{14}\times\frac{4}{-8}=\frac{-1}{2}\times\frac{-1}{2}$ (By writing in standard form)

$$=\frac{(-1)\times(-1)}{2\times2}=\frac{1}{4}$$

(ii) $\frac{16}{-20}\times\frac{3}{-9}=\frac{-4}{5}\times\frac{-1}{3}$ (By writing in standard form)

$$=\frac{(-4)\times(-1)}{5\times3}=\frac{4}{15}$$

***Example 21:*** Find the product of the following.

(i) $7\times\frac{-3}{21}$ (ii) $\frac{-14}{26}\times3$ (iii) $-8\times\frac{10}{40}$

***Solution:***

(i) $7\times\frac{-3}{21}=\frac{7}{1}\times\frac{-1}{7}=\frac{7\times(-1)}{1\times7}=\frac{-7}{7}=-1$

(ii) $\frac{-14}{26}\times3=\frac{-7}{13}\times\frac{3}{1}=\frac{(-7)\times3}{13\times1}=\frac{-21}{13}$

(iii) $-8\times\frac{10}{40}=\frac{-8}{1}\times\frac{1}{4}=\frac{(-8)\times1}{1\times4}=\frac{-8}{4}=-2$



## Reciprocal of rational numbers

Just as in the case of a fraction, reciprocal of a rational number can be found by reversing it. Hence, in a reciprocal the numerator becomes the denominator and the denominator becomes the numerator.

For example,

(i) The reciprocal of $\frac{-6}{13}$ is $\frac{13}{-6}\left(\text{or }\frac{-13}{6}\right)$.

(ii) The reciprocal of $\frac{-3}{8}$ is $\frac{8}{-3}\left(\text{or }\frac{-8}{3}\right)$.

(iii) The reciprocal of $-4$ is $\frac{1}{-4}\left(\text{or }\frac{-1}{4}\right)$.

**Quick Tip**

The reciprocal of a negative rational number is also negative.

## Division of rational numbers

Let us now learn about the rules for division of rational numbers.

1. Write both the rational numbers in their standard form.
2. Find the reciprocal of the divisor.
3. Use the rules of multiplication of rational numbers and multiply the dividend by the reciprocal of the divisor.
4. Write the quotient in the standard form.

For example, $\frac{-6}{7}\div\frac{3}{-14}=\frac{-6}{7}\div\frac{(-3)}{14}$

$$=\frac{-6}{7}\times\frac{14}{-3}=\frac{(-6)\times14}{7\times(-3)}=4$$

## Solved Examples

***Example 22:*** Divide the following.

(i) $\frac{-6}{7}$ by $\frac{2}{-3}$ (ii) $\frac{5}{-11}$ by $\frac{-17}{22}$

***Solution:***

(i) $\frac{-6}{7}\div\frac{2}{-3}=\frac{-6}{7}\div\frac{-2}{3}=\frac{-6}{7}\times\frac{3}{-2}=\frac{-6}{7}\times\frac{-3}{2}$

$$=\frac{(-6)\times(-3)}{7\times2}=\frac{18}{14}=\frac{9}{7}$$

(ii) $\frac{5}{-11}\div\frac{-17}{22}=\frac{-5}{11}\div\frac{-17}{22}=\frac{-5}{11}\times\frac{22}{-17}$

$$=\frac{-5}{11}\times\frac{-22}{17}=\frac{(-5)\times(-22)}{11\times17}=\frac{110}{187}=\frac{10}{17}$$

***Example 23:*** By which number should we multiply $\frac{6}{-7}$ to get $\frac{1}{2}$?

***Solution:*** Let us take a simple example. By which number should we multiply 2 to get 6? We know that the answer is 3. How do we get it? By dividing 6 by 2.

Similarly in this question, the required number will be $\frac{1}{2}\div\frac{6}{-7}$.

$$\frac{1}{2}\div\frac{6}{-7}=\frac{1}{2}\times\frac{-7}{6}=\frac{1\times(-7)}{2\times6}=\frac{-7}{12}$$

Let us verify this by multiplying $\frac{6}{-7}$ and $\frac{-7}{12}$.

$$\frac{6}{-7}\times\frac{-7}{12}=\frac{(-6)\times(-7)}{(-7)\times12}=\frac{1}{2}$$

Thus, the required number is $\frac{-7}{12}$.

***Example 24:*** The product of three rational numbers is $\frac{-4}{5}$. If two of the numbers are $\frac{1}{-2}$ and $\frac{3}{5}$, find the third number.

***Solution:*** The third number will be $\frac{-4}{5}\div\left(\frac{1}{-2}\times\frac{3}{5}\right)$

$$\frac{-4}{5}\div\left(\frac{1}{-2}\times\frac{3}{5}\right)=\frac{-4}{5}\div\left(\frac{-1}{2}\times\frac{3}{5}\right)=\frac{-4}{5}\div\left(\frac{(-1)\times3}{2\times5}\right)$$

$$=\frac{-4}{5}\div\left(\frac{-3}{10}\right)=\frac{-4}{5}\times\frac{10}{-3}=\frac{-4}{5}\times\frac{-10}{3}$$

$$=\frac{(-4)\times(-10)}{5\times3}=\frac{40}{15}=\frac{8}{3}$$

**Check:**

$$\frac{1}{-2}\times\frac{3}{5}\times\frac{8}{3}=\frac{-1}{2}\times\frac{3}{5}\times\frac{8}{3}=\frac{(-1)\times3\times8}{2\times5\times3}=\frac{-24}{30}=\frac{-4}{5}$$

Thus, the third number is $\frac{8}{3}$.

## EXERCISE 1.3

1. Add the following rational numbers.

(i) $\frac{6}{-7}+\frac{-1}{7}+\frac{-3}{7}$ (ii) $\frac{4}{11}+\frac{-6}{11}+\frac{-3}{-11}$

(iii) $\frac{4}{7}+\frac{-2}{3}$ (iv) $\frac{-7}{11}+\frac{-1}{5}$

(v) $\frac{2}{-3}+\frac{-7}{12}+\frac{5}{6}$ (vi) $\frac{1}{4}+\frac{-3}{-8}+\frac{7}{12}$

2. Subtract the following.

(i) $\frac{3}{16}$ from $\frac{-8}{16}$ (ii) $\frac{-9}{15}$ from $\frac{-6}{15}$

(iii) $\frac{11}{-13}$ from $\frac{1}{2}$ (iv) $\frac{-9}{15}$ from $\frac{-2}{3}$

(v) $\frac{5}{11}$ from $\frac{-18}{22}$ (vi) $\frac{13}{16}$ from $\frac{17}{24}$

3. Find the value of the following.

(i) $\frac{4}{7}-\frac{-1}{3}+\frac{8}{21}$ (ii) $\frac{2}{5}+\frac{-7}{-10}-\frac{8}{15}$

(iii) $\frac{11}{12}+\frac{-3}{-8}+\frac{-1}{4}$ (iv) $\frac{5}{6}-\frac{-3}{4}-\frac{-9}{12}$

(v) $\frac{3}{5}-\frac{-6}{-10}+\frac{-8}{15}$ (vi) $\frac{-1}{-4}-\frac{11}{12}+\frac{-7}{8}$

4. Find the following products.

(i) $\frac{(-7)}{6}\times(-2)$ (ii) $\frac{11}{(-22)}\times\frac{3}{4}$

(iii) $\frac{1}{(-9)}\times\frac{2}{(-7)}\times 3$ (iv) $\frac{3}{(-5)}\times\frac{(-10)}{3}$

(v) $\frac{3}{(-4)}\times\frac{3}{(-4)}\times 16$ (vi) $\frac{(-7)}{9}\times\frac{2}{3}\times\frac{(-9)}{14}$

5. Write the reciprocal of the following.

(i) $\frac{9}{-11}$ (ii) $\frac{-5}{-6}$ (iii) $-3$

(iv) $\frac{16}{-21}$ (v) $\frac{-10}{-25}$ (vi) $\frac{3}{-39}$

6. Divide the following.

(i) $\frac{4}{5}$ by $\frac{-1}{6}$ (ii) $\frac{-3}{-7}$ by $\frac{6}{14}$

(iii) $\frac{7}{-14}$ by $\frac{1}{-2}$ (iv) $\frac{6}{-13}$ by $(-5)$

(v) $(-11)$ by $\frac{1}{7}$ (vi) $\frac{100}{-200}$ by 5

7. Evaluate the following.

(i) $\left\{\frac{(-2)}{3}+\frac{1}{2}\right\}\div\left\{\frac{(-4)}{5}+2\right\}$

(ii) $\left\{\frac{(-5)}{6}\div\frac{1}{3}\right\}\div\left\{\frac{2}{3}\times\frac{-6}{5}\right\}$

(iii) $\left\{\frac{3}{(-5)}\times(-7)\right\}\times\left\{\frac{4}{7}\div\frac{2}{-7}\right\}$

(iv) $\left\{(-8)\times\frac{1}{5}\right\}-\left\{\frac{6}{11}\div\frac{1}{-11}\right\}$

(v) $\left\{\frac{4}{-7}+\frac{-9}{7}\right\}\div\left\{\frac{3}{5}\times\frac{(-10)}{(-15)}\right\}$

8. Which is greater—the sum of $\frac{-7}{10}$ and $\frac{3}{-7}$ or the sum of $\frac{-1}{8}$ and $\frac{15}{24}$? By how much?

9. Add the product of $\frac{-2}{5}$ and $\frac{1}{3}$ to the product of $\frac{-3}{16}$ and $\frac{4}{-15}$.

10. Subtract the quotient of $\left(\frac{-4}{5}\div\frac{1}{5}\right)$ from the quotient of $\left(\frac{2}{3}\div\frac{-6}{9}\right)$.



## Properties of Fundamental Operations

In the previous section, we have learnt how to carry out the four fundamental operations on integers and rational numbers. In this section, let us learn about the properties of these operations.

### Commutative property

This property states that if an operation is performed over two numbers $a$ and $b$, then the order of two numbers is not important, that is, whether we take $a$ first or $b$ first does not matter.

Let us check this property for the operations of addition, subtraction, multiplication and division on various sets of numbers.

**Commutative property for addition of numbers**

1. Natural numbers
   We see that $6+4=10$ and $4+6=10$.
2. Whole numbers
   We see that $0+8=8$ and $8+0=8$.
3. Integers
   We see that $(-7)+(-2)=(-9)$ and $(-2)+(-7)=(-9)$.
4. Rational numbers
   We see that $\left(\frac{-1}{2}\right)+\left(\frac{3}{4}\right)=\frac{(-2)+3}{4}=\frac{1}{4}$ and
   $\left(\frac{3}{4}\right)+\left(\frac{-1}{2}\right)=\frac{3+(-2)}{4}=\frac{1}{4}$

In all the above cases, we see that the answers are same irrespective of the order of numbers.

Thus, commutative property is true for addition of numbers.

$\Rightarrow \quad a+b=b+a$

**Commutative property for subtraction of numbers**

1. Natural numbers
   We see that $9-8=1$ but $8-9\neq 1$.
2. Whole numbers
   We see that $9-0=9$ but $0-9\neq 9$.
3. Integers
   We see that $(-6)-(-4)=-2$ but $(-4)-(-6)\neq -2$.
4. Rational numbers
   We see that $\left(\frac{1}{2}\right)-\left(\frac{6}{22}\right)=\frac{11-6}{22}=\frac{5}{22}$ and
   $\left(\frac{6}{22}\right)-\left(\frac{1}{2}\right)=\frac{6-11}{22}=\frac{-5}{22}$

In all the above cases, we see that if we change the order of numbers, the answers are different.

Thus, commutative property is not true for subtraction of numbers.

$\Rightarrow \quad a-b\neq b-a$

**Commutative property for multiplication of numbers**

1. Natural numbers
   We see that $6\times 7=42$ and $7\times 6=42$.
2. Whole numbers
   We see that $0\times 100=0$ and $100\times 0=0$.
3. Integers
   We see that $(-2)\times(-3)=6$ and $(-3)\times(-2)=6$.
4. Rational numbers
   We see that $\left(\frac{-1}{2}\right)\times\left(\frac{3}{4}\right)=\left(\frac{-3}{8}\right)$ and
   $\left(\frac{3}{4}\right)\times\left(\frac{-1}{2}\right)=\left(\frac{-3}{8}\right)$

In all the above cases, we see that the answers are same irrespective of the order of numbers.

Thus, commutative property is true for multiplication of numbers.

$\Rightarrow \quad a\times b=b\times a$

**Commutative property for division of numbers**

1. Natural numbers
   We see that $6\div 3=2$ but $3\div 6\neq 2$.
2. Whole numbers
   We see that $0\div 11=0$ but $11\div 0$ is not defined.
3. Integers
   We see that $(-14)\div 7=-2$ but $7\div(-14)\neq -2$.

4. Rational numbers

We see that $\left(\frac{1}{2}\right) \div \left(\frac{-4}{10}\right) = \frac{-5}{4}$ and

$\left(\frac{-4}{10}\right) \div \left(\frac{1}{2}\right) \neq \frac{-5}{4}$

In all the cases, we see that if we change the order of numbers, the answers are different.

Thus, commutative property is not true for division of numbers.

$\Rightarrow \quad a \div b \neq b \div a$

To summarise the **commutative** property, we can say that:

| Numbers | Commutative for | | | |
|---|---|---|---|---|
| | Addition | Subtraction | Multiplication | Division |
| Natural | Yes | No | Yes | No |
| Whole | Yes | No | Yes | No |
| Integers | Yes | No | Yes | No |
| Rational | Yes | No | Yes | No |

Therefore, according to commutative property,

$a + b = b + a$ and $a \times b = b \times a$

## Solved Examples

***Example 25:*** Verify commutative property for addition and multiplication of the following numbers.

(i) 6 and 5 (ii) 0 and 3

(iii) (–7) and 5 (iv) $\frac{-3}{5}$ and $\frac{7}{10}$

***Solution:*** If we have two numbers, $a$ and $b$, then according to:

(a) Commutative property for addition

$a + b = b + a$

(b) Commutative property for multiplication

$a \times b = b \times a$

(i) $6 + 5 = 11$ and $5 + 6 = 11$

$6 \times 5 = 30$ and $5 \times 6 = 30$

Hence verified.

(ii) $0 + 3 = 3$ and $3 + 0 = 3$

$0 \times 3 = 0$ and $3 \times 0 = 0$

Hence verified.

(iii) $(-7) + 5 = -2$ and $5 + (-7) = -2$

$(-7) \times 5 = -35$ and $5 \times (-7) = -35$

Hence verified.

(iv) $\left(\frac{-3}{5}\right) + \frac{7}{10} = \left(\frac{-6}{10}\right) + \frac{7}{10} = \frac{1}{10}$ and

$\frac{7}{10} + \left(\frac{-3}{5}\right) = \frac{7}{10} + \left(\frac{-6}{10}\right) = \frac{1}{10}$

$\left(\frac{-3}{5}\right) \times \frac{7}{10} = \left(\frac{-3 \times 7}{5 \times 10}\right) = \frac{-21}{50}$ and

$\frac{7}{10} \times \left(\frac{-3}{5}\right) = \left(\frac{7 \times -3}{10 \times 5}\right) = \frac{-21}{50}$

Hence verified.

***Example 26:*** Solve both parts and compare the answers. Are they same?

(i) (a) $\frac{11}{(-20)} - \frac{1}{4}$ (b) $\frac{1}{4} - \left\{\frac{11}{(-20)}\right\}$

(ii) (a) $\frac{2}{3} \div \left(\frac{-7}{9}\right)$ (b) $\left(\frac{-7}{9}\right) \div \frac{2}{3}$

Does commutative property hold good for subtraction and division of numbers?



***Solution:***

(i) (a) $\frac{11}{(-20)} - \frac{1}{4} = \left(\frac{-11}{20}\right) - \frac{1}{4} = \left(\frac{-11}{20}\right) - \frac{5}{20}$

$= \frac{-16}{20} = \frac{-4}{5}$

(b) $\frac{1}{4} - \left\{\frac{11}{(-20)}\right\} = \frac{1}{4} - \left(\frac{-11}{20}\right) = \frac{1}{4} + \frac{11}{20}$

$= \frac{5}{20} + \frac{11}{20} = \frac{16}{20} = \frac{4}{5}$

We can see that $\frac{11}{(-20)} - \frac{1}{4} \neq \frac{1}{4} - \left\{\frac{11}{(-20)}\right\}$.

(ii) (a) $\frac{2}{3} \div \left(\frac{-7}{9}\right) = \frac{2}{3} \times \left(\frac{-9}{7}\right) = \left(\frac{2 \times (-9)}{3 \times 7}\right) = \frac{-18}{21} = \frac{-6}{7}$

(b) $\left(\frac{-7}{9}\right) \div \frac{2}{3} = \left(\frac{-7}{9}\right) \times \frac{3}{2} = \left(\frac{(-7) \times 3}{9 \times 2}\right) = \frac{-21}{18} = \frac{-7}{6}$

We can see that $\frac{2}{3} \div \left(\frac{-7}{9}\right) \neq \left(\frac{-7}{9}\right) \div \frac{2}{3}$.

Hence, the commutative property does not hold true for subtraction and division of numbers.

## Associative property

This property states that if an operation is performed on a group of three numbers $a$, $b$ and $c$, the order of grouping of these numbers is not important. Any grouping brings about the same result.

Let us check this property for the operations of addition, subtraction, multiplication and division on various sets of numbers.

### Associative property for addition of numbers

**1.** Natural numbers

$(3 + 4) + 5 = 7 + 5 = 12$

$3 + (4 + 5) = 3 + 9 = 12$

**2.** Whole numbers

$(0 + 9) + 6 = 9 + 6 = 15$

$0 + (9 + 6) = 0 + 15 = 15$

**3.** Integers

$(-6 + 4) + (-7) = -2 + (-7) = -9$

$(-6) + \{4 + (-7)\} = -6 + (-3) = -9$

**4.** Rational numbers

$\left(\frac{-4}{5} + \frac{3}{5}\right) + \frac{2}{5} = \left(\frac{-4+3}{5}\right) + \frac{2}{5} = \frac{-1}{5} + \frac{2}{5} = \frac{1}{5}$

$\left(\frac{-4}{5}\right) + \left(\frac{3}{5} + \frac{2}{5}\right) = \frac{-4}{5} + \frac{5}{5} = \frac{1}{5}$

In all the above cases, we see that the answers are same irrespective of the order of numbers.

Thus, associative property is true for addition of numbers.

$\Rightarrow \quad (a + b) + c = a + (b + c)$

### Associative property for subtraction of numbers

**1.** Natural numbers

$(3 - 4) - 1 = -1 - 1 = -2$

$3 - (4 - 1) = 3 - 3 = 0$

**2.** Whole numbers

$(6 - 0) - 7 = 6 - 7 = -1$

$6 - (0 - 7) = 6 - (-7) = 13$

**3.** Integers

$\{-6 - (-3)\} - 5 = \{-6 + 3\} - 5 = -3 - 5 = -8$

$(-6) - \{(-3 - 5)\} = -6 - (-8) = -6 + 8 = 2$

**4.** Rational numbers

$\left(\frac{-1}{2} - \frac{2}{3}\right) - \frac{5}{6} = \left(\frac{-3-4}{6}\right) - \frac{5}{6} = \frac{-7}{6} - \frac{5}{6} = \frac{-12}{6}$

$= -2$

$\frac{-1}{2} - \left(\frac{2}{3} - \frac{5}{6}\right) = \frac{-1}{2} - \left(\frac{4-5}{6}\right) = \frac{-1}{2} - \frac{-1}{6}$

$= \frac{-3-(-1)}{6} = \frac{-2}{6} = \frac{-1}{3}$

In all the above cases, we see that if we change the order of numbers, the answers are different.

Thus, associative property is not true for subtraction of numbers.

$\Rightarrow \quad (a - b) - c \neq a - (b - c)$

### Associative property for multiplication of numbers

**1.** Natural numbers

$(2 \times 3) \times 5 = 6 \times 5 = 30$

$2 \times (3 \times 5) = 2 \times 15 = 30$

2. Whole numbers

$(0 \times 4) \times 6 = 0 \times 6 = 0$

$0 \times (4 \times 6) = 0 \times 24 = 0$

3. Integers

$\{(-5) \times (-4)\} \times 6 = 20 \times 6 = 120$

$(-5) - \{(-4) \times 6)\} = (-5) \times (-24) = 120$

4. Rational numbers

$\left(\frac{-2}{7} \times \frac{1}{3}\right) \times \frac{-4}{5} = \frac{-2}{21} \times \frac{-4}{5} = \frac{8}{105}$

$\frac{-2}{7} \times \left(\frac{1}{3} \times \frac{-4}{5}\right) = \frac{-2}{7} \times \frac{-4}{15} = \frac{8}{105}$

In all the above cases, we see that the answers are same irrespective of the order of numbers.

Thus, associative property is true for multiplication of numbers.

$\Rightarrow \quad (a \times b) \times c = a \times (b \times c)$

### Associative property for division of numbers

1. Natural numbers

$(6 \div 3) \div 2 = 2 \div 2 = 1$

$6 \div (3 \div 2) = 6 \div \frac{3}{2} = 6 \times \frac{2}{3} = 4$

2. Whole numbers

$(10 \div 5) \div 2 = 2 \div 2 = 1$

$10 \div (5 \div 2) = 10 \div \frac{5}{2} = 10 \times \frac{2}{5} = 4$

3. Integers

$\{(-6) \div 7\} \div 5 = \frac{-6}{7} \div 5 = \frac{-6}{7} \times \frac{1}{5} = \frac{-6}{35}$

$-6 \div (7 \div 5) = -6 \div \frac{7}{5} = -6 \times \frac{5}{7} = \frac{-30}{7}$

4. Rational numbers

$\left(\frac{-1}{3} \div \frac{4}{5}\right) \div \frac{6}{11} = \left(\frac{-1}{3} \times \frac{5}{4}\right) \div \frac{6}{11} = \frac{-5}{12} \times \frac{11}{6} = \frac{-55}{72}$

$\frac{-1}{3} \div \left(\frac{4}{5} \div \frac{6}{11}\right) = \frac{-1}{3} \div \left(\frac{4}{5} \times \frac{11}{6}\right) = \frac{-1}{3} \div \frac{22}{15}$

$= \frac{-1}{3} \times \frac{15}{22} = \frac{-5}{22}$

In all the above cases, we see that if we change the order of numbers, the answers are different.

Thus, associative property is not true for division of numbers.

$\Rightarrow \quad (a \div b) \div c \neq a \div (b \div c)$

To summarise the **associative** property, we can say that:

| Number | Associative for | | | |
|---|---|---|---|---|
| | Addition | Subtraction | Multiplication | Division |
| Natural | Yes | No | Yes | No |
| Whole | Yes | No | Yes | No |
| Integers | Yes | No | Yes | No |
| Rational | Yes | No | Yes | No |

Therefore, according to associative property,

$(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$

## SOLVED EXAMPLES

**Example 27:** Verify associative property for addition and multiplication of the following numbers.

(i) 6, 3 and 4 (ii) 0, 2 and 9

(iii) (–2), 5 and (–3) (iv) $\frac{4}{5}, \frac{6}{5}$ and $\frac{-7}{5}$

**Solution:** If we have three numbers, $a$, $b$ and $c$, then according to:

(a) Associative property for addition

$(a + b) + c = a + (b + c)$

(b) Associative property for multiplication

$(a \times b) \times c = a \times (b \times c)$

(i) $(6 + 3) + 4 = 9 + 4 = 13$ and

$6 + (3 + 4) = 6 + 7 = 13$



$(6 \times 3) \times 4 = 18 \times 4 = 72$ and

$6 \times (3 \times 4) = 6 \times 12 = 72$

Hence verified.

(ii) $(0 + 2) + 9 = 2 + 9 = 11$ and

$0 + (2 + 9) = 0 + 11 = 11$

$(0 \times 2) \times 9 = 0 \times 9 = 0$ and

$0 \times (2 \times 9) = 0 \times 18 = 0$

Hence verified.

(iii) $(-2 + 5) + (-3) = 3 + (-3) = 0$

and $(-2) + \{5 + (-3)\} = (-2) + 2 = 0$

$(-2 \times 5) \times (-3) = (-10) \times (-3) = 30$ and

$(-2) \times \{5 \times (-3)\} = (-2) \times (-15) = 30$

Hence verified.

(iv) $\left(\frac{4}{5} + \frac{6}{5}\right) + \left(\frac{-7}{5}\right) = \frac{10}{5} + \left(\frac{-7}{5}\right) = \frac{3}{5}$ and

$\frac{4}{5} + \left\{\frac{6}{5} + \left(\frac{-7}{5}\right)\right\} = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

$\left(\frac{4}{5} \times \frac{6}{5}\right) \times \left(\frac{-7}{5}\right) = \frac{24}{25} \times \left(\frac{-7}{5}\right) = -\frac{168}{125}$ and

$\frac{4}{5} \times \left\{\frac{6}{5} \times \left(\frac{-7}{5}\right)\right\} = \frac{4}{5} \times \left(\frac{-42}{25}\right) = \frac{-168}{125}$

Hence verified.

**Example 28:** Solve both parts and compare the answers. Are they same?

(i) (a) $\left(\frac{9}{11} - \frac{4}{11}\right) - \frac{6}{9}$ (b) $\frac{9}{11} - \left(\frac{4}{11} - \frac{6}{9}\right)$

(ii) (a) $\left(\frac{11}{17} \div \frac{6}{17}\right) \div \frac{3}{8}$ (b) $\frac{11}{17} \div \left(\frac{6}{17} \div \frac{3}{8}\right)$

Does associative property hold true for subtraction and division of numbers?

**Solution:**

(i) (a) $\left(\frac{9}{11} - \frac{4}{11}\right) - \frac{6}{9} = \frac{5}{11} - \frac{2}{3} = \frac{15}{33} - \frac{22}{33} = \frac{-7}{33}$

(b) $\frac{9}{11} - \left(\frac{4}{11} - \frac{6}{9}\right) = \frac{9}{11} - \left(\frac{-10}{33}\right) = \frac{9}{11} + \frac{10}{33} = \frac{19}{33}$

We can see that $\left(\frac{9}{11} - \frac{4}{11}\right) - \frac{6}{9} \neq \frac{9}{11} - \left(\frac{4}{11} - \frac{6}{9}\right)$

(ii) (a) $\left(\frac{11}{17} \div \frac{6}{17}\right) \div \frac{3}{8} = \left(\frac{11}{17} \times \frac{17}{6}\right) \div \frac{3}{8} = \frac{187}{102} \times \frac{8}{3} = \frac{44}{9}$

(b) $\frac{11}{17} \div \left(\frac{6}{17} \div \frac{3}{8}\right) = \frac{11}{17} \div \left(\frac{6}{17} \times \frac{8}{3}\right)$

$= \frac{11}{17} \div \frac{48}{51} = \frac{11}{17} \times \frac{51}{48} = \frac{33}{48}$

We can see that $\left(\frac{11}{17} \div \frac{6}{17}\right) \div \frac{3}{8} \neq \frac{11}{17} \div \left(\frac{6}{17} \div \frac{3}{8}\right)$

Hence, the associative property does not hold true for subtraction and division of numbers.

### Distributive property

Let us now learn about the distributive property of multiplication over addition and subtraction.

### Distributive property of multiplication over addition

If there are three numbers $a$, $b$ and $c$, then

$$a \times (b + c) = a \times b + a \times c$$

Let us check this property on various sets of numbers.

1. Natural and whole numbers

We see that $2 \times (5 + 8) = 2 \times 13 = 26$ and

$2 \times 5 + 2 \times 8 = 10 + 16 = 26$

2. Integers

We see that $(-5) \times (3 + 4) = (-5) \times 7 = -35$ and

$(-5) \times 3 + (-5) \times 4 = -15 - 20 = -35$

3. Rational numbers

We see that

$\left(\frac{-2}{5}\right) \times \left(\frac{3}{4} + \frac{1}{2}\right) = \left(\frac{-2}{5}\right) \times \left(\frac{5}{4}\right) = \frac{-10}{20} = \frac{-1}{2}$ and

$\left(\frac{-2}{5}\right) \times \frac{3}{4} + \left(\frac{-2}{5}\right) \times \frac{1}{2} = \frac{-6}{20} + \left(\frac{-2}{10}\right) = \frac{-10}{20} = \frac{-1}{2}$

Thus, multiplication is distributive over addition for all sets of numbers.

**Distributive property of multiplication over subtraction**

If there are three numbers $a$, $b$ and $c$, then

$$a \times (b - c) = a \times b - a \times c$$

Let us check this property on various sets of numbers.

1. Natural and whole numbers

   We see that $2 \times (8 - 5) = 2 \times 3 = 6$

   and $2 \times 8 - 2 \times 5 = 16 - 10 = 6$

   Hence, both the values are same. However, if the first number inside the bracket is smaller than the second, say '5 – 8' we cannot say this as we cannot find the value of $2 \times (5 - 8)$.

2. Integers

   We see that $(-5) \times (3 - 4) = (-5) \times -1 = 5$ and $(-5) \times 3 - (-5) \times 4 = -15 + 20 = 5$

3. Rational numbers

   We see that $\left(\frac{-2}{5}\right) \times \left(\frac{3}{4} - \frac{1}{2}\right) = \left(\frac{-2}{5}\right) \times \left(\frac{1}{4}\right)$

   $= \frac{-2}{20} = \frac{-1}{10}$ and

   $\left(\frac{-2}{5}\right) \times \frac{3}{4} - \left(\frac{-2}{5}\right) \times \frac{1}{2} = \frac{-6}{20} - \left(\frac{-2}{10}\right) = \frac{-2}{20} = \frac{-1}{10}$

Thus, multiplication is not always distributive over subtraction for natural and whole numbers but it is always distributive over subtraction for integers and rational numbers.

## Solved Examples

***Example 29:*** Verify the following.

(i) $-3 \times (4 - 5) = -3 \times 4 - (-3) \times 5$

(ii) $\frac{4}{7} \times \left(\frac{5}{6} + \frac{1}{3}\right) = \frac{4}{7} \times \frac{5}{6} + \frac{4}{7} \times \frac{1}{3}$

***Solution:***

(i) LHS $= -3 \times (4 - 5) = -3 \times (-1) = 3$

RHS $= -3 \times 4 - (-3) \times 5 = -12 + 15 = 3$

LHS = RHS = 3

Hence verified.

(ii) LHS

$= \frac{4}{7} \times \left(\frac{5}{6} + \frac{1}{3}\right) = \frac{4}{7} \times \left(\frac{5}{6} + \frac{2}{6}\right) = \frac{4}{7} \times \left(\frac{7}{6}\right) = \frac{2}{3}$

RHS

$= \frac{4}{7} \times \frac{5}{6} + \frac{4}{7} \times \frac{1}{3} = \frac{10}{21} + \frac{4}{21} = \frac{14}{21} = \frac{2}{3}$

LHS = RHS $= \frac{2}{3}$

Hence verified.

***Example 30:*** Solve both parts and compare the answers. Are they same?

(i) (a) $\frac{6}{9} \times \left(\frac{9}{11} - \frac{4}{11}\right)$ (b) $\frac{6}{9} \times \frac{9}{11} - \frac{6}{9} \times \frac{4}{11}$

(ii) (a) $\frac{3}{8} \times \left(\frac{11}{17} + \frac{6}{17}\right)$ (b) $\frac{3}{8} \times \frac{11}{17} + \frac{3}{8} \times \frac{6}{17}$

***Solution:***

(i) (a) $\frac{6}{9} \times \left(\frac{9}{11} - \frac{4}{11}\right) = \frac{6}{9} \times \frac{5}{11} = \frac{10}{33}$

(b) $\frac{6}{9} \times \frac{9}{11} - \frac{6}{9} \times \frac{4}{11} = \frac{6}{11} - \frac{8}{33} = \frac{10}{33}$

We can see that $\frac{6}{9} \times \left(\frac{9}{11} - \frac{4}{11}\right) = \frac{6}{9} \times \frac{9}{11} - \frac{6}{9} \times \frac{4}{11}$

(ii) (a) $\frac{3}{8} \times \left(\frac{11}{17} + \frac{6}{17}\right) = \frac{3}{8} \times \frac{17}{17} = \frac{3}{8}$

(b) $\frac{3}{8} \times \frac{11}{17} + \frac{3}{8} \times \frac{6}{17} = \frac{33}{136} + \frac{18}{136} = \frac{51}{136} = \frac{3}{8}$

We can see that $\frac{3}{8} \times \left(\frac{11}{17} + \frac{6}{17}\right) = \frac{3}{8} \times \frac{11}{17} + \frac{3}{8} \times \frac{6}{17}$



## Exercise 1.4

1. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $0 + 8$ (b) $8 + 0$
   - (ii) (a) $6 - 3$ (b) $3 - 6$
   - (iii) (a) $0 - 4$ (b) $4 - 0$
   - (iv) (a) $(-5) - (-2)$ (b) $(-2) - (-5)$

2. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $\frac{2}{7} + \frac{1}{3}$ (b) $\frac{1}{3} + \frac{2}{7}$
   - (ii) (a) $\frac{4}{13} + \frac{2}{13}$ (b) $\frac{2}{13} + \frac{4}{13}$
   - (iii) (a) $\frac{4}{9} - \frac{1}{2}$ (b) $\frac{1}{2} - \frac{4}{9}$
   - (iv) (a) $\frac{-9}{13} - \frac{4}{13}$ (b) $\frac{4}{13} - \frac{-9}{13}$

3. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $6 \times 4$ (b) $4 \times 6$
   - (ii) (a) $(-5) \times (-2)$ (b) $(-2) \times (-5)$
   - (iii) (a) $3 \div 8$ (b) $8 \div 3$
   - (iv) (a) $(-9) \div (-3)$ (b) $(-3) \div (-9)$

4. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $\frac{6}{11} \times \frac{-2}{7}$ (b) $\frac{-2}{7} \times \frac{6}{11}$
   - (ii) (a) $\frac{13}{-6} \div \frac{7}{18}$ (b) $\frac{7}{18} \div \frac{13}{-6}$
   - (iii) (a) $\frac{5}{9} \times \frac{-9}{15}$ (b) $\frac{-9}{15} \times \frac{5}{9}$
   - (iv) (a) $\frac{11}{13} \div \frac{4}{-39}$ (b) $\frac{4}{-39} \div \frac{11}{13}$

5. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $\{(-2) + 5\} + 7$ (b) $-2 + (5 + 7)$
   - (ii) (a) $(8 - 3) - 2$ (b) $8 - (3 - 2)$
   - (iii) (a) $(5 - 0) - 7$ (b) $5 - (0 - 7)$
   - (iv) (a) $\{(-8) - 4\} - 5$ (b) $(-8) - (4 - 5)$

6. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $\left(\frac{6}{12} + \frac{9}{6}\right) + \frac{2}{3}$ (b) $\frac{6}{12} + \left(\frac{9}{6} + \frac{2}{3}\right)$
   - (ii) (a) $\left(\frac{4}{7} + \frac{2}{7}\right) + \frac{1}{10}$ (b) $\frac{4}{7} + \left(\frac{2}{7} + \frac{1}{10}\right)$
   - (iii) (a) $\left(\frac{7}{12} - 0\right) - \frac{4}{9}$ (b) $\frac{7}{12} - \left(0 - \frac{4}{9}\right)$
   - (iv) (a) $\left(\frac{2}{11} - \frac{6}{11}\right) - \frac{2}{3}$ (b) $\frac{2}{11} - \left(\frac{6}{11} - \frac{2}{3}\right)$

7. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $(6 \times 1) \times 4$ (b) $6 \times (1 \times 4)$
   - (ii) (a) $[(-2) \times 5] \times 3$ (b) $(-2) \times (5 \times 3)$
   - (iii) (a) $(14 \div 7) \div 2$ (b) $14 \div (7 \div 2)$
   - (iv) (a) $\{(-8) \div 4\} \div (-8)$ (b) $(-8) \div \{4 \div (-8)\}$

8. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $\left(\frac{6}{7} \times \frac{4}{18}\right) \times \frac{1}{2}$ (b) $\frac{6}{7} \times \left(\frac{4}{18} \times \frac{1}{2}\right)$
   - (ii) (a) $\left(\frac{11}{16} \times \frac{-5}{11}\right) \times \frac{2}{-3}$ (b) $\frac{11}{16} \times \left(\frac{-5}{11} \times \frac{2}{-3}\right)$
   - (iii) (a) $\left(\frac{4}{6} \div \frac{1}{2}\right) \div \frac{-4}{6}$ (b) $\frac{4}{6} \div \left(\frac{1}{2} \div \frac{-4}{6}\right)$
   - (iv) (a) $\left(\frac{-7}{8} \div \frac{6}{7}\right) \div \frac{4}{7}$ (b) $\frac{-7}{8} \div \left(\frac{6}{7} \div \frac{4}{7}\right)$

9. Solve both the parts and compare the answers. Are they the same?
   - (i) (a) $7 \times (22 - 4)$ (b) $7 \times 22 - 7 \times 4$
   - (ii) (a) $(-3) \times (5 + 2)$ (b) $(-3) \times 5 + (-3) \times 2$

(iii) (a) $(-11) \times \{6-(-2)\}$

(b) $(-11) \times 6-(-11) \times (-2)$

(iv) (a) $7 \times \{(-2)-(-12)\}$

(b) $7 \times (-2)-7 \times (-12)$

**10.** Solve both the parts and compare the answers. Are they the same?

(i) (a) $\frac{7}{3} \times \left(\frac{2}{9}-\frac{3}{4}\right)$ (b) $\frac{7}{3} \times \frac{2}{9}-\frac{7}{3} \times \frac{3}{4}$

(ii) (a) $\left(\frac{-11}{3}\right) \times \left(\frac{5}{22}+\frac{2}{11}\right)$

(b) $\left(\frac{-11}{3}\right) \times \frac{5}{22}+\left(\frac{-11}{3}\right) \times \frac{2}{11}$

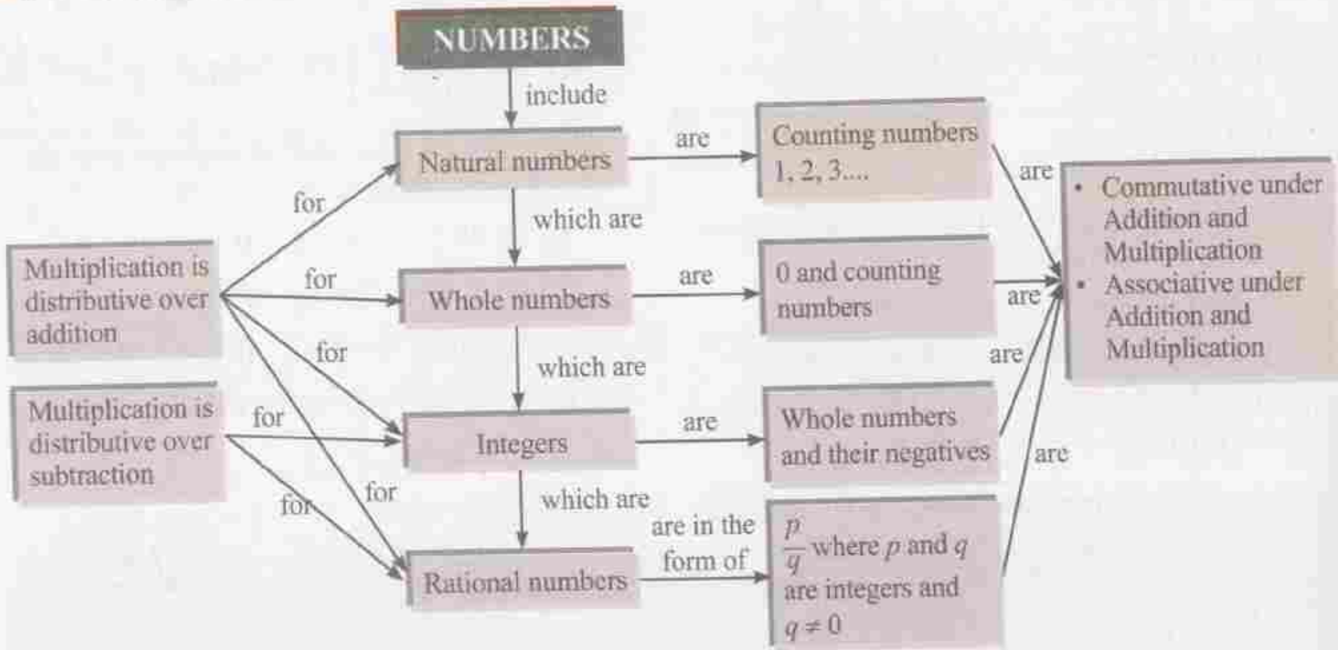
(iii) (a) $\frac{4}{5} \times \left\{\frac{6}{5}-\left(\frac{-2}{5}\right)\right\}$

(b) $\frac{4}{5} \times \frac{6}{5}-\frac{4}{5} \times \left(\frac{-2}{5}\right)$

(iv) (a) $\left(\frac{-7}{2}\right) \times \left\{\left(\frac{-2}{3}\right)-\left(\frac{-12}{7}\right)\right\}$

(b) $\left(\frac{-7}{2}\right) \times \left(\frac{-2}{3}\right)-\left(\frac{-7}{2}\right) \times \left(\frac{-12}{7}\right)$

## Quick Recall




## Revision Exercise

**1.** Write the following as rational numbers.

(i) $-4$ (ii) $3.49$ (iii) $0$ (iv) $16$

**2.** Write the following rational numbers in standard form.

(i) $\frac{-16}{-80}$ (ii) $\frac{7}{-63}$ (iii) $\frac{120}{-360}$ (iv) $\frac{19}{95}$

**3.** Evaluate the following.

(i) $\{(-6) \times (-2)\} \div \{(-1) \times (-4)\}$

(ii) $\{10-8+(-4)\} \times \{16 \div (-8)\}$

(iii) $\{17+(-5)-(-6)\} \div \{(-9) \times (-2)\}$

(iv) $\{18-20+(-6)\} \times \{(-8) \div (-1)\}$

**4.** The product of three integers is $(-1000)$. If two of the integers are $(-8)$ and $(-125)$, find the third integer.



**5.** Find the value of:

(i) $\frac{6}{7}-\frac{1}{3}-\frac{5}{21}$ (ii) $\frac{-8}{11}+\frac{3}{11}-\frac{17}{33}$

(iii) $\frac{-2}{5}+\frac{1}{10}-\frac{7}{15}$ (iv) $\frac{5}{8}-\frac{1}{4}+\frac{-7}{12}$

**6.** Divide the following.

(i) $\frac{-7}{11}$ by $\frac{14}{22}$ (ii) $\frac{19}{23}$ by $\frac{2}{23}$

(iii) $\frac{-6}{7}$ by $\frac{11}{21}$ (iv) $\frac{16}{-25}$ by $\frac{4}{5}$

**7.** Add the product of $\frac{-9}{14}$ and $\frac{7}{3}$ to the product of $\frac{-2}{3}$ and $\frac{6}{7}$.

**8.** Divide $\frac{-16}{19}$ by $\frac{64}{-76}$ and add the quotient to the sum of $\frac{-2}{3}$ and $\frac{5}{6}$.

**9.** Fill in the box by the correct number.

(i) $\frac{-4}{6}+\frac{-3}{7}=\square+\frac{-4}{6}$

(ii) $(-6) \times [(-3) \times (-4)]=[(-6) \times \square] \times (-4)$

(iii) $\left(\frac{-3}{11}\right) \times \frac{5}{6} \times \square=\frac{5}{6} \times \frac{-3}{11} \times \frac{17}{20}$

(iv) $(-6)+(-7)+(-11)=\square+(-6)+(-7)$

(v) $\frac{4}{3} \times \left(\frac{3}{5}-\square\right)=\square \times \frac{3}{5}-\frac{4}{3} \times \frac{5}{6}$

(vi) $\square \times \left(\frac{2}{5}-\frac{9}{4}\right)=\left(\frac{-7}{13}\right) \times \frac{2}{5}-\left(\frac{-7}{13}\right) \times \frac{9}{4}$

**10.** Evaluate the following.

(i) $\left(\frac{9}{12} \times \frac{-24}{27}\right) \times \left(\frac{8}{11} \times \frac{33}{40}\right)$

(ii) $\left(\frac{4}{7} \times \frac{28}{40}\right) \times \left(\frac{-5}{13} \times \frac{26}{30}\right)$

(iii) $\left(\frac{-5}{13} \times \frac{26}{30}\right) \times \left(\frac{5}{7} \times \frac{21}{50}\right)$

(iv) $\left(\frac{1}{-2} \times \frac{4}{7}\right) \times \left(\frac{18}{23} \times \frac{-46}{90}\right)$

**11.** Verify the following.

(i) $(-6) \times \{4+(-3)\}=(-6) \times 4+(-6) \times (-3)$

(ii) $(-3) \times \{(-10)-(-6)\}$
$=(-3) \times (-10)-(-3) \times (-6)$

(iii) $\frac{4}{5} \times \left\{\frac{3}{7}-\left(\frac{-1}{2}\right)\right\}=\frac{4}{5} \times \frac{3}{7}-\frac{4}{5} \times \left(\frac{-1}{2}\right)$

(iv) $\left(\frac{-9}{5}\right) \times \left\{\left(\frac{-5}{18}\right)-\frac{10}{9}\right\}$
$=\left(\frac{-9}{5}\right) \times \left(\frac{-5}{18}\right)-\left(\frac{-9}{5}\right) \times \frac{10}{9}$

## Face the Challenge

**1.** Get number 24 by only using the numbers 8, 8, 3, 3. You can use the signs of addition, subtraction, multiplication and division.

**2.** Fill in the boxes with appropriate numbers.

(i) $\square \xrightarrow{+\frac{2}{4}} \square \xrightarrow{\times\frac{-1}{2}} \square \xrightarrow{\div\frac{5}{-2}} \boxed{\frac{-6}{11}}$

(ii) $\square \xrightarrow{-\frac{-1}{2}} \square \xrightarrow{\times\frac{3}{4}} \square \xrightarrow{+\frac{2}{3}} \boxed{\frac{41}{24}}$

Create two more such statements with boxes for your friend to solve.